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AERODYNAMIC CHARACTERISTICS OF VOLUME ROD CONSTRUCTIONS IN STATIONARY FREE-MOLECULAR FLOW*

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The flow of a high-velocity current around a volume rod construction (RC) /1/ - that is, a spatial structure consisting of a large number of cylindrical rods fastened at their ends, and length of each one of which is significantly less than the overall size of the construction, is considered well-known concepts /2, 3/ are used.

It is established that, to a first approximation, the resistance of an RC, the average over the cell of cell properties which are the same for all the cells, is proportional to the volume of the RC, if by this we mean the volume of the polyhedron whose edges are the axes of the exterior rods. It is explained that the aerodynamic moment acting on such an RC with respect to its centre of mass is equal to zero. A number of RCs with a high degree of symmetry in the arrangement of the rods in the cells are investigated. For these, we discover the absence of a transverse force in the case of local-diffusional scattering of the molecules by portions of the rod surfaces.

1. Rod constructions (RCs), which enable us to construct light pieces of astronautical apparatus of significant dimensions, are spatial configurations consisting of a large number N of cylindrical rods, fastened at the ends, of diameter $d_i \sim d$, and length $l_i \sim l$, $1 \leqslant i \leqslant N$, where $d \ll l \ll L$, where L is the characteristic dimension of the construction overall (Fig.1).

When a free-molecular current flows part an RC /2/, an aerodynamic shadow is formed behind each rod, which spreads gradually as a result of thermal motion of the molecules. We distinguish a "strong" shadow zone, where the velocity head is close to zero, and a "weak" shadow zone, where the velocity head is of the same order as in the unperturbed current. The distance g at which the final spreading of the strong shadow occurs is $g \approx \mathbf{M}\, d/2$, where $\mathbf{M} \gg 1$ is the Mach number.

Under orbital flight conditions $g \ll l$, that is, the strong shadows of the rods spread to a distance much less than the characteristic dimension of the rods. As a consequence of this, strong mutual shading of the rods can only occur at portions adjacent to the rod attachment corners, that is, on a relatively small part of the rod surface. Therefore, the influence of strong shadowing on the aerodynamic characteristics of the RC is insignificant. It can be proved that the relative error on neglecting strong shadowing does not exceed $O(\mathbf{M}d/l) \ll 1$, and the average value (over all possible orientations of the RC in the current) of this error is $O(d/l) \ll 1$. In this paper, we do not take account of strong shadowing, and the indicated terms of the estimate are omitted everywhere below.

In weak mutual shadowing of the rods, the expanding and spreading shadow of each rod falls, as a rule, on a large number of rods located downstream, if there are such rods. And, contrariwise, on each rod there fall, as a rule, the shadows of a large number of rods that are located upstream, if there are such rods. In other words, weak mutual shadowing of the rods leads to a gradual attenuation of the current in the RC. Weak shadowing will be considered below.

To describe the local properties of the RC, we introduce an intermediate dimension: Λ: $l \ll \Lambda \ll L$. Let P_R be a sphere of radius Λ centred at the points R.

We define the thickness $\varkappa$ of the RC at the point R as the ratio of the total length of the axes of the rods arranged in P_R to the volume of P_R. It is obvious that the thickness $\varkappa$ does not depend on the diameters of the rods located in P_R, and

$$\varkappa = Kl^{-2}$$

where K is a coefficient whose value is the same for structures possessing local geometric similarity.

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We will consider in more detail some of the simplest "regular" structures: 4-ray and 6-ray structures (Fig.2a, b, respectively). Four rods at different angles to each other emerge from each vertex of the 4-ray structure; 6 rays, directed pairwise along three mutually perpendicular **axes,** emerge from each vertex of the 6-ray structure. The **length of** each rod of the structures is the same and equal to l, and we will also take the diameter to be the same and equal to d. We call a subset of rods oriented the same way as family. The four-ray structure is the union of four rod families, and the 6-ray structure is a union of three families. A calculation gives $K = 3^{3/2}/4$ for the 4-ray structure, and $K = 3.$ for the 6-ray structure.

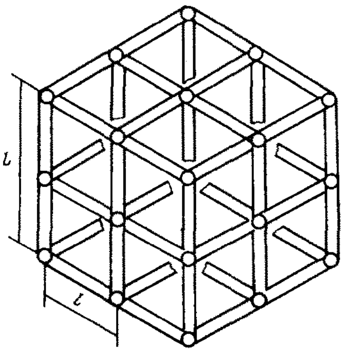

Fig.1

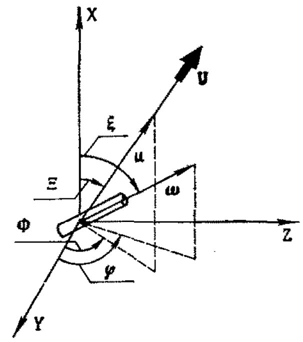

Fig.3

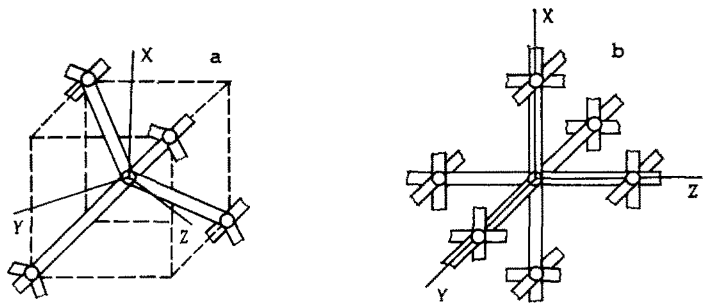

Fig.2

2. We obtain an expression for that part of the RC resistance which is due to the effect of incident molecules (superscript +) of the incoming current. Considering a homogeneous RC (one for which all the properties obtained, like the thickness $\varkappa$, by averaging over P_R, do not depend on the position of the point R) we explain that for it this part of the resistance coefficient is given by a power series in the parameter $\varepsilon = dL/l^2$. Here, the leading term of the series is linear in ε.

Everywhere below, we will mean by the volume of an RC the volume of the polyhedron whose edges are the axes of the external rods of the RC. Here, we will assume that no fewer than three rods that do not lie in the same plane emerge from any vertex-otherwise, the polyhedron will have degenerate two-dimensional fragments.

Let XYZ be a Cartesian system of coordinates connected with the RC and centred at R - an internal point of the RC. To describe the orientation of the rods in P_R and the current directions, we introduce a spherical system of coordinates centred at R (Fig.3). We will measure the angle ξ from the X axis to the direction of the unit vector $\boldsymbol{\omega}$, collinear with the rod axis, and the angle φ from the Y axis to the projection of $\boldsymbol{\omega}$ onto the YZ plane. The unit vector $\mathbf{u} = \mathbf{U}/|\mathbf{U}|$, collinear with the current velocity $\mathbf{U}$, will be specified analogously, measuring the angle Ξ from the X axis to the direction of the vector $\mathbf{u}$, and the angle Φ from the Y axis to the projection of the vector $\mathbf{u}$ on the YZ plane. Since the position of the rod does not change if the direction of $\boldsymbol{\omega}$ is reversed, and the molecule interaction picture is transformed in a centrally-symmetric manner when the direction of the vector $\mathbf{u}$ is reversed, it is sufficient to consider the following region of the angular variables:

$$0 \leqslant \xi \leqslant \frac{\pi}{2}, \quad 0 \leqslant \varphi \leqslant 2\pi, \quad 0 \leqslant \Xi \leqslant \frac{\pi}{2}, \quad 0 \leqslant \Phi \leqslant 2\pi$$

The orientation of the rods in P_R can be described by specifying the probability ∂P that the point T randomly (with a constant linear probability density) chosen from the set of points of the union of the rod axes, arranged in P_R, lies on the rod whose axis is oriented in the angular range $[\xi, \xi + \partial\xi]$, $[\varphi, \varphi + \partial\varphi]$, and the diameter lies in the range $[d, d + \partial d]$:

$$\partial P = f(\xi, \varphi, d)\sin\xi\partial\xi\partial\varphi\partial d \left(\iiint f\sin\xi\partial\xi\partial\varphi\partial d = 1\right)$$

where $f(\xi, \varphi, d)$ is the probability density.

Let η be the local coefficient of RC attenuation of the free-molecular current, which characterizes the relative reduction in the velocity head q when the current travels a distance ∂h: $\partial q/q = -\eta\partial h$. We can ascertain that on neglecting the transverse thermal motion of the molecules the quantity η is equal to the ratio of the total area of the projection (in a plane perpendicular to the current velocity vector) of all the rods located in P_R to the volume of P_R:

$$\eta = \varkappa \iiint sdf\sin\xi\partial\xi\partial\varphi\partial d$$

where $s = s(\xi, \varphi, \Xi, \Phi)$ is the modulus of the projection of the unit vector $\boldsymbol{\omega}$, directed along the ray (ξ, φ), onto the ray perpendicular to the ray (Ξ, Φ):

$$s = [1 - (\cos\xi\cos\Xi + \sin\xi\cos\varphi\sin\Xi\cos\Phi + \sin\xi\sin\varphi\sin\Xi\sin\Phi)^2]^{1/2}$$

We can write the attenuation coefficient in the form

$$\eta = (d/l^2)A(\Xi, \Phi)$$

where d, l are the characteristic diameter and length of the rods; the quantity A is determined by the orientation of the rods relative to the current.

Thus, for a regular 4-ray structure we find

$$A = (3^{1/2}/16)(C_1 + S_1 + C_2 + S_2)$$

$$C_{1,2} = \left[1 - \left(\sqrt{\frac{1}{3}}\cos\Xi \pm \sqrt{\frac{2}{3}}\sin\Xi\cos\Phi\right)^2\right]^{1/2}$$

$$S_{1,2} = \left[1 - \left(\sqrt{\frac{1}{3}}\cos\Xi \pm \sqrt{\frac{2}{3}}\sin\Xi\sin\Phi\right)^2\right]^{1/2}$$

In particular $A = 3^{3/2}2^{-5/2} \approx 0.92$ where the current is incident parallel to some family of rods; $A = 3^{1/2}\pi/16 \approx 1.02$ on average over all the possible orientations of the RC relative to the flow.

For a regular 6-ray structure, we find

$$A = \sin\Xi + (1 - \sin^2\Xi\cos^2\Phi)^{1/2} + (1 - \sin^2\Xi\sin^2\Phi)^{1/2}$$

In particular, $A = 2$ when the current is incident parallel to some family of rods; $A = \sqrt{6} \approx 2.45$ when the current is incident along an axis equally-inclined to the rods; $A = 3\pi/4$ on average over all possible orientations of the RC relative to the current.

To find the velocity head q at a point R located inside the RC, it is necessary to integrate over the linear coordinate h along the straight line passing through R and parallel to the velocity $\mathbf{U}$ of the incoming current, from the point where this line enters the RC up to R. We obtain $q = q_0\exp\left(-\int\eta\partial h\right)$, where q_0 is the high-velocity pressure in the unperturbed current.

We introduce the volume density F_{XA} of the resistance force X_A as the ratio of the total resistance force acting on the rods in P_R to the volume of P_R. Obviously

$$F_{XA}^+ = 2\eta q_0\exp\left(-\int\eta\partial h\right) + O(\eta\, q_0\mathbf{M}^{-2}) \tag{2.1}$$

The error is due to neglecting the thermal motion of the molecules in considering the interaction of the current with the rods.

Integrating (2.1) over the normalized volume of the RC $(V/L^3)_\Sigma$ we obtain for the corresponding part of the RC resistance coefficient

$$C_{XA}^+ = X_A^+/(q_0L^2) =$$

$$2\varepsilon \iiint \left\{A\,|_R \exp\left[-\varepsilon \int A\,|_H\, \partial (h/L)\right]\right\} \partial (V/L^3) + O(\varepsilon \mathbf{M}^{-2}) \tag{2.2}$$

For a homogeneous RC, $A = \text{const}$, and then (2.2) becomes

$$C_{XA}^{+} = 2\varepsilon A \iiint \{\exp[-\varepsilon A (\rho/L)]\}\, \partial (V/L^3) + O(\varepsilon \mathbf{M}^{-2}) \tag{2.3}$$

where ρ is the distance from the entry into the RC of the straight line that contains the point R and is parallel to the velocity of the incoming current, to the point R. Expanding the exponent in (2.3) in a power series in the small parameter $\varepsilon \ll 1$ and integrating term-by-term, we obtain

$$C_{XA}^{+} = 2\varepsilon A\, (V/L^3)(1 - \varepsilon AB) + O(\varepsilon^3, \varepsilon \mathbf{M}^{-2}) \tag{2.4}$$
$$B = V^{-1} \iiint (\rho/L) \partial V$$

where V is the volume of the homogeneous RC, and B is a function of its shape and orientation.

Thus, for a sphere subscript s) of diameter D; a right circular cylinder (subscript cyl) of diameter D and height H and for a right circular cone (subscript con) of diameter D and height H, having a set $L = D$, we find: $B_s = 3/8$; $B_{\text{cyl}} = H/(2D)$, $B_{\text{con}} = H/(4D)$ if the axis of the cylinder (cone) is parallel to the flow; $B_{\text{cyl}} = 4/(3\pi)$, $B_{\text{con}} = 1/\pi$, if the axis is perpendicular to the flow.

We remark that at finite values of the Mach number $\mathbf{M}$, the effect on the RC of the molecules undergoing their first collision may lead to the appearance of a transverse force (perpendicular to the velocity of the incoming current). We can ascertain, however, that the transverse force coefficient is small: $C_\perp = O(\varepsilon \mathbf{M}^{-2})$. Therefore, the transverse force is not taken into account in this paper.

3. We will consider the effect of the force on the RC of the molecules scattered by the rods (superscript -). It was pointed out in /3/ that the velocity of the molecules of the incoming current decreases several-fold after the first collision with a surface if $T_w/T_0 \ll 1$, where T_w is the surface temperature and T_0 is the braking temperature. So the aerodynamic effect component coefficients for the scattered molecules on the RC is of the order of $\varepsilon\beta$, where $\beta = \{\pi\, [(\gamma - 1)/\gamma] T/T_0\}^{1/2}$, $\gamma = c_P/c_V$ is the ratio of the specific heats, and T is the effective temperature of the singly-scattered molecules. On further collisions with the RC rods, the molecules are slowed down still further, and their influence on the aerodynamic characteristics of the RC is completely insignificant. So we neglect them, and also the effect of the current attenuation (the absolute error in the aerodynamic force coefficients arising as a consequence of the neglect of these factors amounts to $O(\varepsilon^2\beta)$).

Then, it is sufficient to describe the dependence of the force on the rod surface on the direction of incidence of the molecules of the incoming flow and to give the distribution of the rods in terms of the diameter and angles of orientation. We will assume that the force of the molecules on a rod surface element depends on the local angle of incidence Θ in the same way as in non-equilibrium locally-diffusional scattering - that is, there is no tangential force effect of the scattered molecules: $c_\tau^- = 0$, the normal force effect coefficient $c_n^- = \beta \cos \Theta$ on the leading side of the rod and $c_n^- = 0$ on the trailing side. To calculate the force effect of the scattered molecules on the rods located in P_R it is necessary to integrate the value of c over the surface of a cylindrical rod of arbitrary orientation, and then over the aggregate of the rods located in P_R.

The longitudinal force (parallel to the rod axis) that acts on the rod from the scattered molecules is not present in locally-diffusional scattering. The transverse force (perpendicular to the rod axis, and lying in the plane passing through the rod axis and the current velocity vector) $\mathbf{f}_\perp^-$ that acts on the rod from the scattered molecules is given /2/ by the expression

$$\mathbf{f}_\perp^- = -\tfrac{1}{4}\pi\beta l d q_0\, [\boldsymbol{\omega} \times [\boldsymbol{\omega} \times \mathbf{u}]] \tag{3.1}$$

Integrating (3.1), we can find the coefficients of the components of the aerodynamic effect on the RC. For the regular 4- and 6-ray structures considered above, we find that the scattered molecules, like the incident ones, do not give rise to a transverse force in the approximation under consideration - that is, up to $O(\varepsilon^2\beta, \varepsilon\mathbf{M}^{-2})$ we have $C_\perp = 0$. Moreover, it turns out that in the approximation under consideration the increase in the resistance force due to the scattered molecules does not depend on the current direction - that is, up to $O(\varepsilon^2\beta, \varepsilon\beta\mathbf{M}^{-2})$ we have $C_{XA}^- = \varepsilon\beta\, (3^{1/2}\pi/8)(V/L^3)$ for a 4-ray structure and $C_{XA}^- = \varepsilon\beta\, (\pi/2)(V/L^3)$ for a 6-ray structure.

4. The aerodynamic moment that acts on the RC is given by the integral over the volume of the RC:

$$\mathbf{M} = \iiint [\mathbf{r} \times \mathbf{F}] \partial V \tag{4.1}$$

where $\mathbf{F}$ is the vector of the volume density of the force effect of the current at the point R with components F_{XA}, F_{YA}, F_{ZA} in the velocity coordinate system $RX_AY_AZ_A$ ($RX_A \parallel \mathbf{U}$; RY_A, $RZ_A \perp \mathbf{U}$ - see /4/); $\mathbf{r}$ is the vector from O to R, relative to which the moment is calculated. We introduce the vector coefficient of the moment as $\mathbf{m} = \mathbf{M}/(q_0L^3)$. It follows from the preceding discussion that

$$\frac{F_{XA}}{q_0L^{-1}} = O(\varepsilon), \quad \left\{\frac{F_{YA}}{q_0L^{-1}}, \frac{F_{ZA}}{q_0L^{-1}}\right\} = O(\varepsilon\beta, \varepsilon\mathbf{M}^{-2}) \tag{4.2}$$

Consequently,

$$m_{XA} = O(\varepsilon\beta, \varepsilon\mathbf{M}^{-2}), \quad \{m_{YA}, m_{ZA}\} = O(\varepsilon)$$

For a homogeneous structure, the terms of the first order in ε on the right-hand sides of (4.2) are constant quantities, and if we move the point O to the geometrical centre of the RC, which corresponds to the centre of mass for a homogeneous RC, then a constant term will give zero on integration. Then for a homogeneous RC

$$m_{XA} = O(\varepsilon^2\beta, \varepsilon^2\mathbf{M}^{-2}), \quad \{m_{YA}, m_{ZA}\} = O(\varepsilon^2)$$

that is, to a first approximation with respect to the parameter ε no aerodynamic moment acts relative to the centre of mass of an homogeneous RC.

We will calculate the magnitude of the aerodynamic moment (in the second approximation with respect to ε) that acts on a homogeneous RC relative to the centre of mass, in the form of a right circular cone with base diameter D and height H, for a flow perpendicular to the axis of symmetry. Let the OX axis coincide with the axis of symmetry and be directed forwards, and let the OY axis be parallel to the flow. We set $L = D$. Substituting (2.4) into (4.1) and carrying out the integration, we find that m_X, $m_Y = 0$ by virtue of the symmetry of the flow, and $m_Z = \varepsilon^2 (A^2/120)(H/D)^2 + O(\varepsilon^2\beta, \varepsilon^2\mathbf{M}^{-2})$.

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